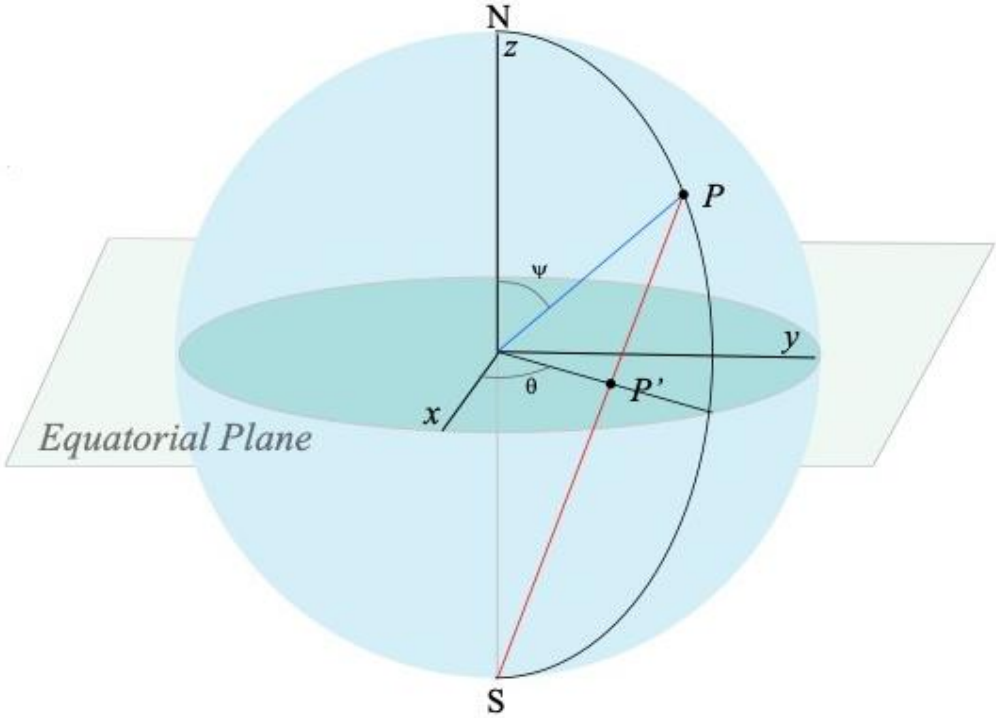
## 1 THE GENERAL STEREOGRAPHIC PROJECTION

A stereographic projection maps circles on the celestial sphere (unit sphere) into circles in the equatorial plane.

Notation:
$\psi$ Polar angle
$\theta$ Azimuth angle
$\phi$ Geographical co-latitude (90˚ – latitude) or zenith distance of the north celestial pole

The position *P* of a point on the sphere is uniquely determined by the angles $\psi$ and $\phi$ (see figure).

*P'* is the stereographically projected point on the equatorial plane. All points on the celestial sphere except the south pole can be mapped onto the equatorial plane. Here we only consider mapping where the north pole is the reference point. The north hemisphere will be mapped outside the equatorial circle, the south hemisphere inside it. Here we use analytical formulas to describe the stereographic mapping, Apian would have used geometrical methods like those described in Ptolemy's Planispherium and explained in Neugebauer'e *History of Ancient Mathematical Astronomy* (Neugebauer, 1975)




The Cartesian coordinates are then given by

$$x = \sin\psi \cos\theta$$
$$y = \sin\psi \sin\theta$$
$$z = \cos\psi$$

In some cases we also want to tilt by an angle $\phi$ around the $y$ axis and then use

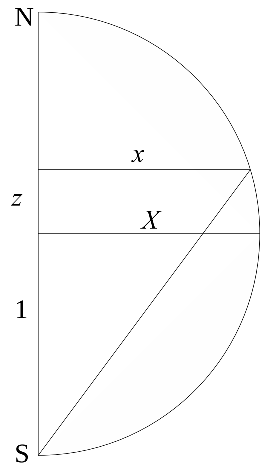
$x' = x \cos \phi - z \sin \phi$

$y' = y$

$z' = x \sin \phi + z \cos \phi$

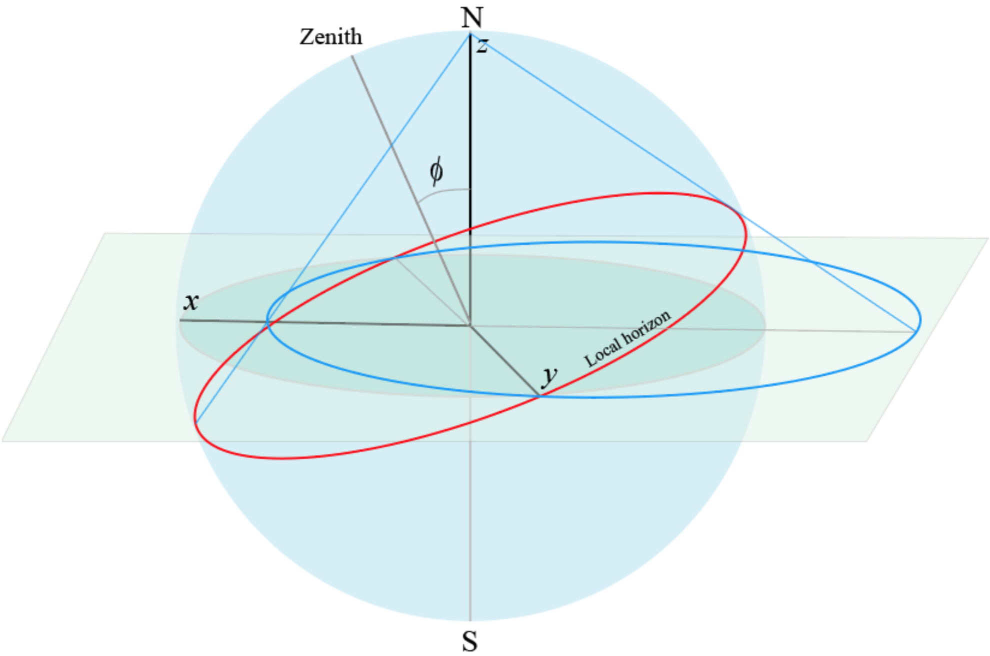
Finally, the stereographically mapped coordinates $X$, $Y$ on the equatorial plane are given by

$X = x' / (1 + z')$ $\qquad$ $Y = y' / (1 + z')$



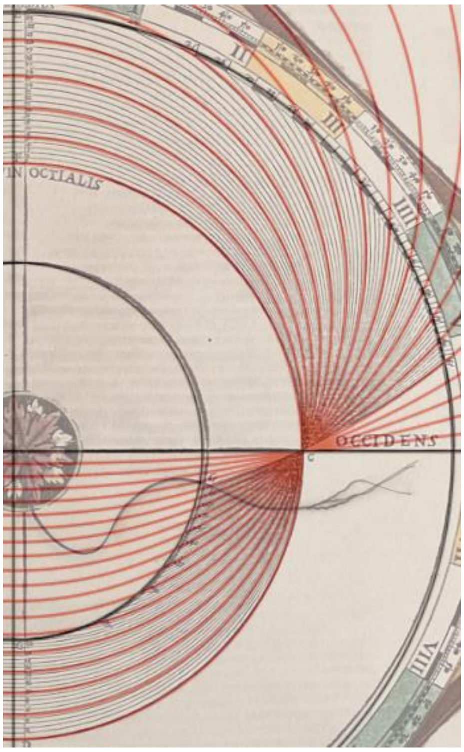

The last relations follow from uniform triangles in the figure. $X$ to 1 as $x$ to 1 + $z$.

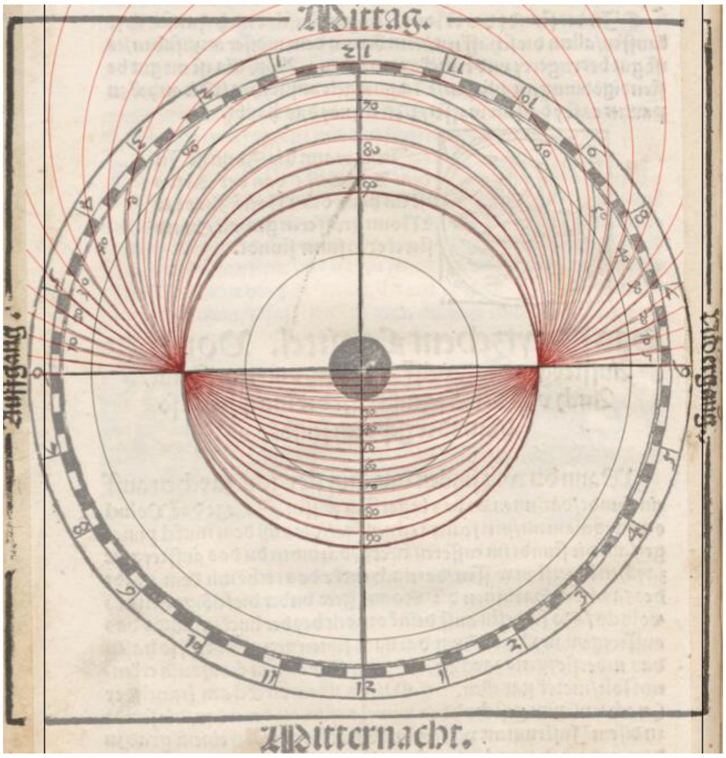
## 2 LOCAL HORIZON CIRCLES




We have $\psi$=90˚ for points in the equatorial plane and project the points with $\theta$ = 0 and 180˚, diametrically opposite points on the horizon circle with a tilt angle $\phi$ being the geographical co-latitude or local zenith distance of the north celestial pole. The Y coordinate will be zero. The distance between the projected points divided by two will give the radius of the projected horizon circle. The difference of the distances from the origin to the projected points, divided by two will give the $X$ coordinate of the center of the horizon circle.

The horizon circles in the volvelle in *Astronomicum Caesareum* agree very well with the computed circles, I show the right halt of the page as the other half of the picture of the book is distorted. The computed circles are red. The horizon lines in *Ein kunstlisch Instrument* agree less well especially for more northern latitudes as seen by the figure.

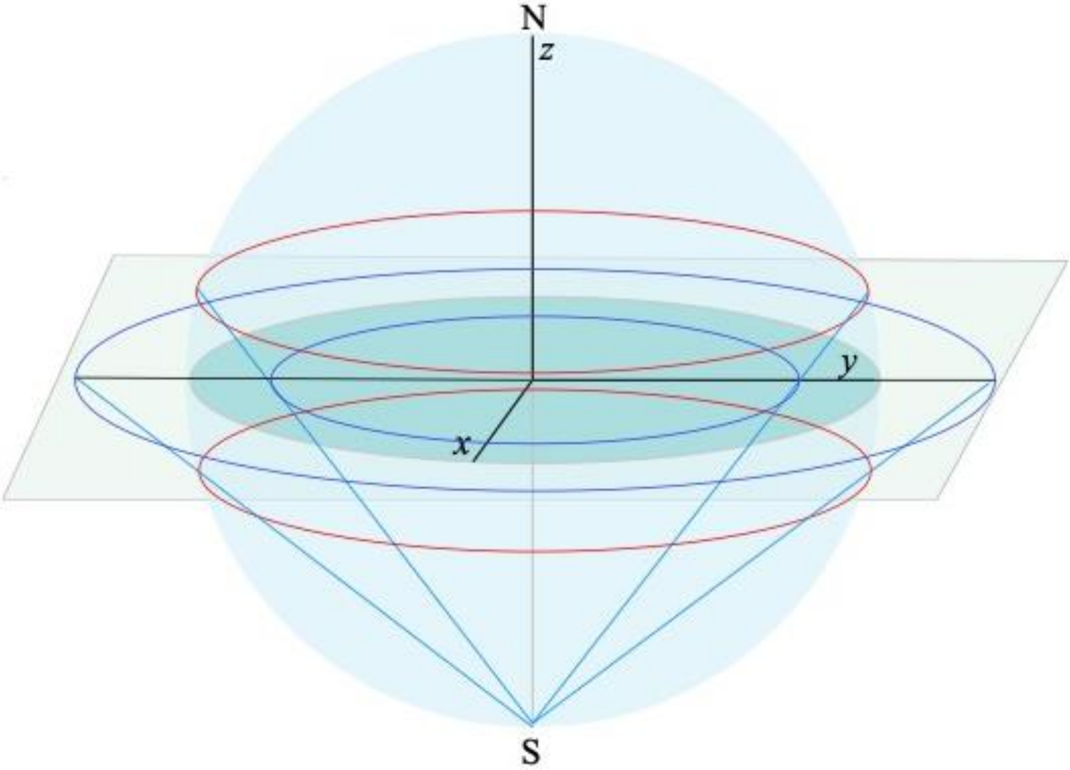



## 3 ECLIPTIC CIRCLE AND LONGITUDE ANGLES

The same procedure is used as for a horizon circle but with $\psi = 90^\circ$ and tilt $\phi = \varepsilon$ where $\varepsilon$ is the obliquity of the ecliptic. The mapped zodiacal longitudes $\Lambda$ of the original longitudes $\lambda$ are computed by putting $\phi = \varepsilon$, and $\theta = \lambda + 90'$ from 0 to $360^\circ$ and $\psi = 90^\circ$.
Only for the cardinal directions these angle agree with the original zodiac angles. As already Ptolemy showed in his Planespherium these angles can be computed by $\tan \alpha = \cos \varepsilon \tan \lambda$ with $\alpha$ and $\lambda$ in the same quadrant. Effectively $\alpha$ will be the right ascension of the ecliptic longitude. This procedure was shown by Ptolemy. The angles agree well with the instrument in *Ein kunstlich Instrument*, see figure where the red lines are computed angles. The ecliptic disk is from the copy in Österreichisches Nationalbibliothek

## 4 PARALLEL CIRCLES




A parallel circle will be mapped into a circle concentric with the origin. The radius will be given by $X$ by taking $\psi = 90^\circ - \delta$, , $\theta = 0$, and $\phi = 0$ where $\delta$ is the declination of the parallel circle. For the tropics $\delta = \pm\, \varepsilon$.

## 5 USING THE VOLVELLE

### 5.1 FINDING SUNRISE AND SUNSET TIMES

The ecliptic overlay is rotated such that the longitude of the Sun at the edge of the zodiac crosses the local co-latitude horizon. The string is stretched over this crossing and the time is read from the time graduation on the periphery of the volvelle. For sites located far north these times will not be accurate due to the neglect of refraction near the horizon.

### 5.2 FINDING THE ASCENDANT AND DESCENDANT

The string is stretched to the hour of the day. The ecliptic overlay is rotated such that the longitude of the Sun is below the string. The rising/ascending sign is then read from where the edge of the zodiac crosses the local co-latitude horizon.

Analytically the longitude of the sign at the horizon is found from the formula

$$\tan \lambda_S = \frac{-\cos \Theta}{\sin \varepsilon \tan \phi + \cos \varepsilon \sin \Theta}$$ (Meeus, Astronomical algorithms, p. 99)

where $\Theta$ is the local sidereal time, that can be computed from $\Theta = H + \alpha$. $H$ is the hour angle of the Sun (time from noon expressed in degrees) and $\alpha$ the right ascension of the Sun given by $\tan \alpha = \cos \varepsilon \tan \lambda$ where $\lambda$ is the longitude of the Sun. Within errors generated by difficulties in the setting and reading of the volvelle, there is an excellent agreement between the analytical formula and the volvelle.

## 6 COMMENTARY

I used a replica with computer generated horizon circles and ecliptic. The volvelle is not easy to handle due to the tightness of the local horizon curves. It is sometimes easier to find the opposite ascending or descending sign. It may have been easier to use the full scale instrument in *Astronomicum Caesareum* which is about 160% larger than the replica but I have a feeling that Apian was not quite happy with the instrument and in the final printing of *Astronomicum Caesareum* the horizon grid was hidden and converted to an instrument for predicting the times of new and full Moons. Another reason for not including the instrument could have been that the book contains rather scanty matter of astrological nature and finally there may not have been enough space for the volvelle.

There is a similar instrument in Apian's book *Ein kunstlich Instrument* of 1524 but where the horizon lines are only mapped in 5° intervals. That instrument was obviously used a lot by readers, in both the extant copies of the book there are hand-written comments and corrections. This instrument was changed from the 1528 editions of *Cosmographia Liber* and on and instead was hidden behind a world map that could be used to find local times at different locations on the earth.. The reason for this may again be that *Cosmographia* contains very little astrology and also that at the time Apian may have found it more interesting to have a volvelle related to the then quite recent discovery of the New World and the circumnavigations of the Earth.

The printing quality of the horizon grid in *Astronomicum Caesareum* is excellent as manifested by the high resolution of the curves. The computational effort behind the curves is enormous and Apian may have used a geometrical method for that. Implementing the result of the computation into the printed result would also have been a very delicate job.

Neugebauer, O. , 1975, *History of Ancient Mathematical Astronomy*, Springer.